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Fundamental Aspects in Quantitative Ultrasonic Determination of Fracture Toughness: The Scattering of a Single Ellipsoidal Inhomogeneity

Li-Sheng W. Fu

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**Fundamental Aspects in Quantitative
Ultrasonic Determination of Fracture
Toughness: The Scattering of a
Single Ellipsoidal Inhomogeneity**

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INTRODUCTION

Advances have been made on a broad front in non-destructive testing (NDT) in terms of measurement methods, instrumentation, automation and computer-assisted signal acquisition and processing while recent developments in fracture mechanics and elastic wave theory have enabled the understanding of many physical phenomena in a mathematical context. The purpose of this review is to bring together the available literature in the material and fracture characterization by NDT, and the related mathematical methods in mechanics that provide fundamental underlying principles for its interpretation and evaluation. Information on the energy release mechanism of defects and the interaction of microstructures within the material is basic in the formulation of the mechanics problems that supply guidance for non-destructive evaluation (NDE).

Since the turn of the century, the growth of NDT has "quietly" taken place. In the methods and technology of NDT all forms of energy such as visual, pressure and leak, penetrant, thermal, magnetic, radiographic, ultrasonic and electromagnetic have been employed to determine all properties of materials. In general, these methods do not directly measure the magnitude of a physical property [1,2]. Much of the work in NDT therefore remain experience-based measurements and await the interpretation and understanding from basic science [3]. Emphasis here in this review is placed on material interrogation by ultrasound and sound.

Only very recently there have been efforts that seek the ties of NDT measurements to the fundamental physical principles in science and

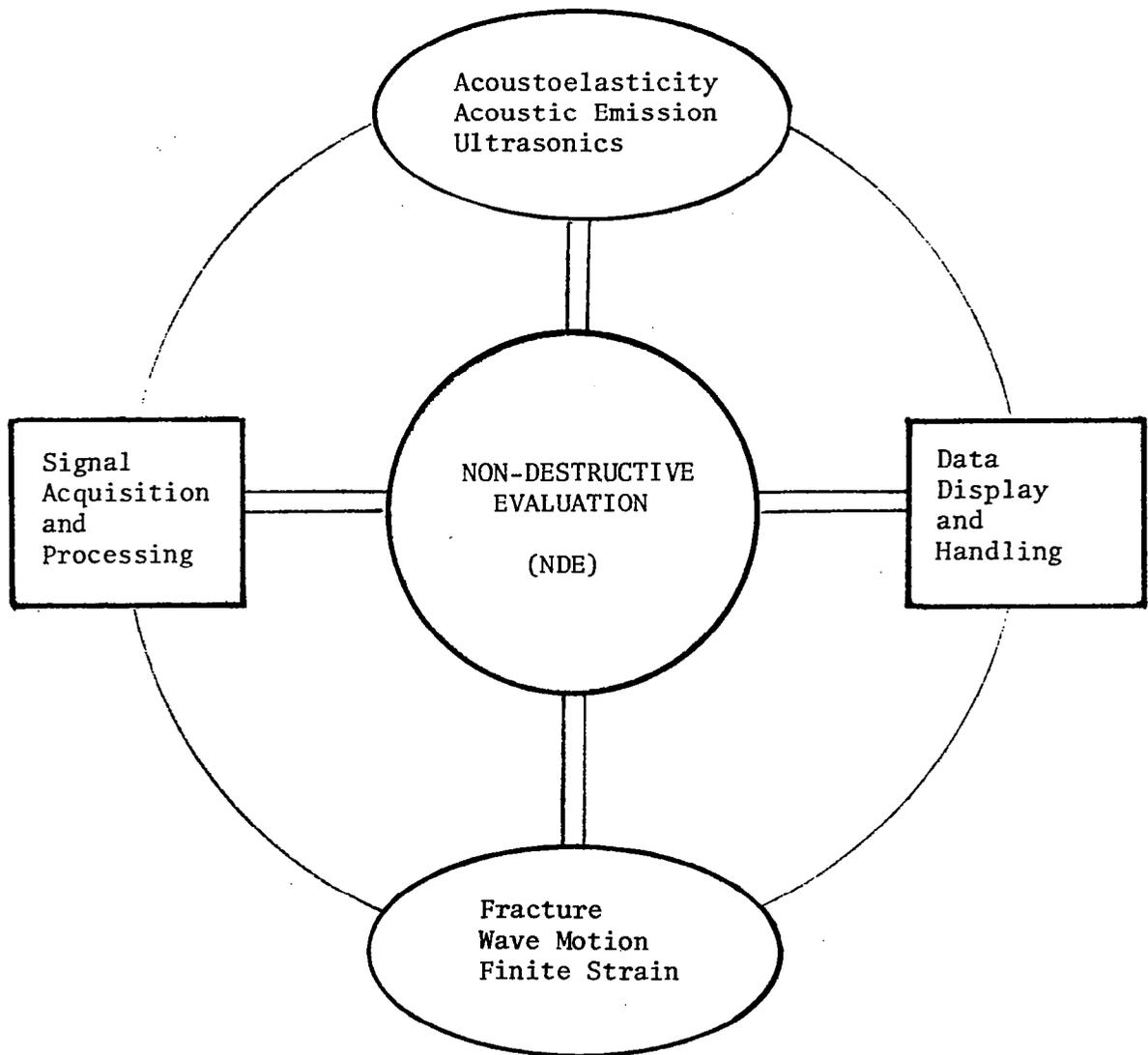


Figure 1. Concept of Non-Destructive Evaluation (NDE).

thereby provide an evaluation of the NDT measurements. The added ingredient of elastic wave propagation and scattering from mechanics and physics transforms the art of NDT to the science of NDE. Early contribution in the material characterization include a few texts from fields such as mechanics, material science and solid state physics [4-9]. A typical concept of NDE is displayed in Figure 1.

While the material characterization by NDT has a long history, the fracture characterization by NDT came about with the recent development of fracture mechanics. The evolution of fracture mechanics through the past forty years or so has led to the establishment of two important concepts: the recognition of a new material property called fracture toughness for ranking materials to be used in design, and the establishment of a new fail-safe design philosophy that requires the retirement of any structural component that contains a crack of critical size. Although the fracture toughness can normally be obtained from carefully designed laboratory mechanical testing, the size and orientation of cracks in structural components must often be measured by non-destructive methods. This has led to a tremendous growth of research activities in elastic wave propagation and NDT methods in crack (flaw) detection and material characterization.

A list of recent texts and survey articles on wave propagation and scattering, fracture mechanics, solid state physics and NDT are listed as Refs. [10-15], Refs. [16-19], Refs. [20,21] and Refs. [22-27], respectively.

In the mechanics literature, NDE by ultrasound is generally discussed under the theory of acoustic emission or acoustoelasticity.

Basically, the speed of wave propagation and energy loss by interaction with material microstructure are the factors obtained for the evaluation and characterization of material properties including fracture toughness. From the fracture mechanics' viewpoint it is important to point out that the criticality of a flaw depends more upon the state of the stress at the crack tip rather than the size of the crack, e.g. a blunted large crack may not propagate. For this reason the stress measurement is included as a part of material characterization here in this review.

The NDE aspect of the reliability and integrity of the structures and materials represents one of the most pressing and critical issues for the coming decade of engineering design and practice, [28-29]. Furthermore, as manufacturing requirements for the primary processing industries often include the need to specify the mechanical and strength properties of materials shipped to customers, a non-destructive on-line measurement of these properties alleviates the problems of testing only a random portion of materials, delay in shipment and cost of mechanical testing. For materials used in e.g. aerospace and nuclear applications, it is very important that fracture toughness be included as a part of the requirement. The non-destructive determination of fracture toughness, however, has not gained much attention until recent years.

In what follows we review material and fracture properties characterization in terms of acoustic emission, acoustoelasticity and ultrasonics. The cut-off time for review is approximately December, 1981. The field encompasses a tremendous volume of material, it is therefore, impossible to include all but selected representative publications.

NDT BY ULTRASOUND--AN OVERVIEW

Frequency Content

Solid state physics and physical acoustics encompass the study of a variety of ultrasonic phenomena involving elastic or stress waves in solids. By convention, ultrasound consists of elastic stress waves within the frequency range from about 20kHz to 1GHz above which it is hypersound. The theoretical estimates of frequency content for acoustic emission events range as high as 50MHz while it is common practice to filter out frequencies below about 5kHz to decrease interference from environmental noise.

Amplitude

The detection of stress waves in materials is usually accomplished with a piezoelectric crystal sensor coupled to the part under test. The same probe is often used in the active mode to introduce ultrasound or acoustic pulses into a material and in the passive mode, between pulses, to receive the resultant echoes. The minimum observable amplitude of the signal will depend on the sensitivity of the detection instrumentation and the level of ambient noise. In acoustic emission, correlations between strains and emission events indicate that the smallest detectable event involves collective movement of five to one hundred and fifty dislocations.

Test Systems

Pulse echo systems are probably the most powerful and universal

tools for ultrasonic measurement of material properties. Foremost among auxiliary ultrasonic methods for this purpose include ultrasonic spectroscopy, resonance testing, acoustic emission, acoustic stimulation and ultrasonic microscopy. The auxiliary methods can be combined with the pulse echo systems or associated with one another in various ways to provide additional information often needed for assessing material property variations [25,27,30].

PHENOMENOLOGICAL
DESCRIPTIONS OF MATERIAL PROPERTIES BY
ULTRASONIC (ACOUSTIC) MEASUREMENTS

The primary thrust of NDE is location and identification of flaws or defects and yet critical angle, velocity, attenuation, acoustic impedance and scattering coefficient can be related to a variety of material constants. Recent conference proceedings are representative of current research [31-34].

Ultrasonic (Acoustic) Velocity

For homogeneous materials, the velocity is simply related to the elastic moduli. In the case of linear elastic, isotropic solid, the longitudinal and transverse velocities, v_L and v_T , respectively, completely determine the elastic moduli with mass density ρ known:

$\lambda + 2\mu = \rho v_L^2$ and $\mu = \rho v_T^2$ where λ and μ are Lamé constants. Because the relations are not linear a careful study must be made of the effect of errors in the measurement of the velocities on the resulted error in the calculations. The velocities are relatively simple to measure with good precision [6,9,22].

Complications arise when the solid is anisotropic or dispersive. In general, the elastic characterization of a material depends on nine independent longitudinal and transverse velocity measurements in three mutually perpendicular directions [6,22]. For uniaxial fibre composite, e.g. five independent velocity measurements will suffice [35,36]. Conversely, given an unknown sample, oriented velocity measurements can be used to determine anisotropy, symmetry, homogeneity, degree of misorientation and similar morphological factors having a bearing on moduli and strength variations. Relative velocity in an acoustic end quench test for steel has been found to be parallel to hardness variation [34,37]. Combination of velocity and Brinell hardness measurements can provide tensile and shear strength of a material [22].

Ultrasonic (Acoustic) Attenuation

As an ultrasonic (acoustic) plane wave passes through a medium, energy is dissipated by a variety of processes. The energy intensity at a distance, x , from the source is given by $I=I_0 \exp(-\alpha x)$ where I_0 is the intensity at $x=0$ and α is the attenuation coefficient, usually expressed as dB per unit distance [6].

The attenuation coefficient consists of two parts, $\alpha=\alpha_a+\alpha_s$, where α_a relates to dislocation movement or any internal friction process and α_s relates to energy scattered away at grain boundaries or other microstructural features. As a rule, attenuation is much more frequency dependent and can be more easily measured relative to frequency than velocity. At frequency range higher than 1MHz scattering losses are

dominant cause of ultrasonic (acoustic) wave attenuation. Among the inhomogeneities that can cause attenuation are precipitates, inclusions, voids, cracks, grain boundaries, interphase boundaries, interstitial impurities, etc. [22].

The only well developed microstructure scattering theories are for grain boundary scattering from equiaxed grains in single phase, polycrystalline solids, scatter attenuation depends on the ratio between average grain size, D , wavelength, λ , and frequency, f . Attenuation mechanisms and regimes are summarized in Table I. Empirical formulas were found to relate grain size to yield strength [38,39] and the change in attenuation was found to be related to early fatigue damage [40] and aging [41].

Ultrasonic Factor: $(v_L \beta_\delta / m)$

Empirical relations between the fracture toughness, K_{IC} , yield strength, Y , and critical attenuation factors, $(v_L \beta_\delta / m)$, related to grain size characterization, δ , were found feasible for two maraging steels and a titanium alloy [42] as $K_{IC}^2 / Y = \psi (v_L \beta_\delta / m)^{1/2}$, and $Y + AK_{IC} + B\beta_1 = C$, where β_δ, β_1 are the attenuation slope measured at $\lambda = \delta$ and $\alpha = 1$, respectively, m, ψ, A, B , are empirical material constants, and λ is wavelength.

From a review based on fracture mechanics literature the feasibility of quantitative ultrasonic determination of fracture toughness was explored in [43]. The underlying principle for using an uncracked specimen and some preliminary data are given in [44]. For an existing

TABLE I

Ultrasonic Attenuation Coefficients for Polycrystalline Alloys

Wavelength range	Attenuation mechanism	Attenuation coefficient
Independent	True absorption	$\alpha_a = C_a f$
$\lambda > D$	Rayleigh scattering	$\alpha_r = C_r D^3 f^4$
$\lambda \leq D$	Phase scattering	$\alpha_p = C_p D f^2$
$\lambda \ll D$	Diffusion scattering	$\alpha_d = C_d / D$

D is "nominal" grain size, λ is wavelength, f is frequency, α is attenuation coefficient, the Cs are experimental constants.

crack, plasticity occurs around the tips of the crack, the characterization of plastic zone and its non-destructive measurement was reviewed and studied in [45]. The plastic zone size at cracking can be related to fracture toughness.

Acoustic Emission

The first scientific report of sounds being emitted from metals was presented in 1928. Acoustic emissions (AE) have been detected in most materials such as wood, glass, concrete, beryllium, fibreglass, ice, many structural metallic alloys, composite materials and honeycomb materials. Conventional displays of acoustic emission data include amplitude of emission (RMS), rate of AE (N), total cumulative count (ΣN) versus other parameters such as load, temperature, fatigue cycles, stress intensity factor, etc. [46-52].

AE has been found to stop during unloading and would not reinitiate until further plastic deformation and fracture occur. The technique has therefore been also used for flaw detection and damage monitoring [53,54].

Acoustoelastic Coefficients

Sound velocity in solids is found to be dependent upon the state of stress in the medium. The acoustoelastic coefficient B depends on the second and third order elastic constants. When these coefficients are measured with sufficient accuracy, the stresses can be obtained by measuring the acoustic velocity at the stressed and unstressed states. The acoustoelastic coefficients are microstructure sensitive material properties [9,55,64,65,66].

Summary

The phenomenological description and characterization of material and fracture properties have led to much research and applications in a variety of fields such as material science, nuclear and aerospace engineering, seismology, semiconductor, etc. Table II gives a review of the material and fracture characterization by ultrasonic (acoustic) methods.

TABLE II

Examples of Material and Fracture Property Characterization
by Ultrasound (Acoustic) Methods

Material property	Applicable material	Measurement technique	Reference
Elastic moduli	Structural solids, single crystals, polycrystals	Velocity methods resonance methods	[6,9,23]
	Ceramics, brittle metals		[56,57]
	Cryogenic metals		[58]
	Refractory metals	Rod wave velocity method	[36]
	Fiber composite laminates	Attenuation and velocities	[59]
Tensile and shear strength	Brittle materials, cast metals	Longitudinal, transverse velocity methods	[22]
	Fiber composites	Acoustic stimulation	[60]
Interlaminar shear strength	Fiber composite laminates	Velocity and attenuation method	[61]
Cohesive strength	Adhesive bonds	Resonance method	[62]
		Spectrum analysis method	[63]
Yield strength	Polycrystalline metals	Resonance method	[42,39]
	Carbon Steel	Wideband attenuation method	
Impact strength	Polycrystalline metals	Narrow bandwidth attenuation method	[23]
Fracture toughness	Polycrystalline metals	Wideband attenuation methods	[42]
Hardness	Polycrystalline metals	Attenuation and velocity methods	[22]
Acoustoelastic coefficient	Polycrystalline metals	Atenuation and velocity methods	[9,55,64 65,66]
Residual stress	Polycrystalline metals	Acoustical holography	[67]
Surface characterization	Polycrystalline metals	Frequency analysis	[68,69]

MECHANICS ASPECTS OF THE
NON-DESTRUCTIVE EVALUATION BY
SOUND AND ULTRASOUND

It appears that earlier impacts on the non-destructive evaluation of materials by employing the theory of elastic wave motion were made by the solid state physicists, [4,6,8,9]. Along with the developments with elastodynamics [10-12, 71,72], mechanics have begun to make major contributions in the application of elastodynamics to NDT by sound and ultrasound and other fields such as electrical devices, seismology, fracture technology, etc. [32,33,70,73,74].

Considerable advances have been made in the direct scattering of a defect (inhomogeneity, void or flaw) and certain amount of work is available in the inverse problem for NDE of crack size and orientation. Theoretical analyses of AE signals in crack detection and material evaluation are available. There are basically three methods used to solve the NDE related problems of elastic wave diffraction and scattering. These works are reviewed under the titles of ray theory, transition matrix and integral equations.

There have appeared, recently, work on NDE of applied stresses and residual stresses in connection to non-linear elasticity. This is reviewed under the title of acoustoelasticity.

Elastic Wave Diffraction and Scattering

The deviation of the wave from its original path is known as the diffraction [10], and the radiation of secondary waves from an embedded obstacle is called the scattering. The obstacle may be in the form of a crack, a rigid body or a second phase particle with different moduli

from that of the surrounding. The problem of an elastic scatterer is more difficult than that of a vacancy or a rigid inclusion in that both the displacements and tractions at the interface are unknowns.

Historically, many theories of scattering, until a few years ago, dealt with scalar waves and simple obstacles. Within that context, two regimes are apparently distinct from each other, the long wavelength regime and the short wavelength or imaging regime. The treatment of vector fields in the elastic solids is much more complex than classical fields of scalar waves and electromagnetic waves. Several useful techniques are, however, extended from these classical fields. Three such methods useful at least beyond the Rayleigh limit were briefly sketched in [43] and an excellent survey and review of the mathematical theories and analysis was given by Pao [75]. Other critical summaries can be found in [33,13,73].

(1) Ray theory

At high frequencies the diffraction of elastic waves by obstacles can be analyzed on the basis of elastodynamic ray theory. For time harmonic wave motion, ray theory provides a method to trace the amplitude of a disturbance as it propagates along a ray. The theory is based on the use of certain canonical exact solutions, for example, the Kirchoff solution for an edge, the diffraction of a plane wave by a semi-infinite crack, etc. These canonical solutions are appropriately adjusted to account for curvatures of incident wave fronts, edges and finite dimensions. The pertinent canonical solution must first be obtained.

The technique of geometrical diffraction theory was introduced by Keller [76] for scalar waves. The extension of this ray theory to the diffraction of elastic waves by smooth obstacles and cracks was first studied by Resende [77] and subsequently by Achenbach and Gautesen [78]. Problems attacked include the direct diffraction and scattering of penny-shaped cracks [78-80], surface breaking cracks [81,82] and the construction of inversion procedures that relate to experimental measurements based on Kirchoff approximation [83] and Fermat principle [84].

A different ray theory was developed for the study of transient elastic waves in multi-layered solid [85,86], in medium bounded by spherical and cylindrical surfaces [87,88] and in inhomogeneous media [89]. The construction of the ray integrals and how these integrals can be evaluated in closed form by using the Cagniard method [90,12,15] can be found in [15] and in a review given by Pao and Gajewski [91]. Arrival times of generalized rays and numerical evaluation of the complex integrals were also discussed. Transient waves generated by point source(s) in an infinite plate and that on the surface of a plate were analyzed by Pao, et. al. [92] to study the mechanism of the source of acoustic emission. The analysis is based on the generalized ray theory and Cagniard's method. Rhodes and Sachse [93] reported measurements on arrival times of ultrasonic pulses scattered by solid inclusions in various matrices and showed how the general ray theory can be used to relate the arrival times of the various scattered signals to the size and wave speed of the inclusion.

(2) Transition Matrix Method

The general theory of the scattering of acoustic waves is contained in the mathematical theory of Huygen's principle [94]. The scattered waves outside the scatterer are related to the Helmholtz integral over the surface of the scatterer. Waterman was first to introduce the transition matrix (T-matrix) method for acoustic scattering [95] and subsequently [96] for electromagnetic waves. Starting from the Helmholtz integral formula, he expanded both the incident and scattered waves in series of the basis functions. He showed that the unknown coefficients of the scattered waves are related by the transition matrix to the coefficient of the incident waves. This approach was applied, in 1976, in the scattering of elastic waves by Waterman [97] and Varatharajulu and Pao [98]. An excellent summary of how the method can be used for acoustic waves and elastic waves is given by Pao [99]. Based on the Betti's third identity, he derived the transition matrix for scattered elastic waves by an inclusion of arbitrary shape.

The T-matrix has infinite number of elements. These elements are integrals of the basis functions over the bounding surface of the scatterer and depend upon the incident wave frequency, geometry of the scatterer and the material properties. The integrals are numerically evaluated. The algorithm developed by Waterman [100] was used to give numerical results of the scattering of compressive and shear waves by a single smooth obstacle in [101,102].

In employing the T-matrix for numerical results, the cost for computing time is increasingly more expansive as the wave number ka becomes large or when the obstacle becomes crack like. In principle,

the T-matrix approach does not give solution for a sharp crack without special treatment for the crack tip. Weaver and Pao [103] reported results for the diffraction of SH-waves by a crack of length $2a$.

(3) Method of Integral Equation and Integral Representation

The theory developed by Fredholm [104,105] for the solution of certain types of linear integral equations has been well-known and widely used in mathematical physics and mechanics. The main reason being at least twofold: (1) if the kernel is separable, the problem of solving an integral equation of the second kind reduces to that of solving an algebraic system of equations. Any reasonably well-behaved kernel can be written as an infinite series of degenerate kernels [106]. (2) The Fredholm theorems provide an assurance of the existence of the solution hence approximate methods can be applied with confidence when the integral equation cannot be solved in close form [105].

This method was applied to problems in quantum scattering first by physicist in a discussion of the convergence of the Born series and by Reinhardt and Szabo [107] in suggesting a numerical procedure for elastic scattering by constructing the Fredholm determinant which contains all the scattering information. The integral representation technique is closely related to the Fredholm integral equation method. Eyges [108] used the integral representation technique to solve the Schrodinger and related equations for irregular and composite regions.

The diffraction of 2-D elastic waves by a crack of finite length has been of interest to investigators in fracture mechanics where the wave field near the crack tip is of main interest [109,110,111]. Coupled

integral equations for displacements potentials are solved numerically and the dynamic stress intensity factors are sought. The time-harmonic excitation of a half plane containing an edge crack has been studied recently by integral transform techniques and reduced to two uncoupled singular integral equations for the symmetric [112] and anti-symmetric [113] scattered fields with respect to the crack plane. The integral equations are numerically solved by using a collocation scheme involving Chebshev polynomials. The crack opening displacements are then calculated.

For the scattering of smooth obstacles or inhomogeneities, it appears that Mal and Knopoff [114] were first in presenting a direct volume integral formulation where they gave the scattered displacements in terms of volume integrals, containing the displacements and strains in the integrand, over the volume of the scatterer. In seeking approximate solutions appropriate at low frequency, a common practice is to use the static displacement field corresponded to the incoming wave fields instead of the actual displacement field. Such an approximation is given in [114] for a perfect spherical inhomogeneity. The same approach was taken by Gubernatis [115] for an ellipsoidal inhomogeneity.

Volume scattering by inhomogeneity or inhomogeneities was also studied by the method of matched asymptotic expansion [116,117] and the polarization approach [118] where results appropriate for the Rayleigh limit were presented. In seeking a connection between the scattering of an inhomogeneity (or inhomogeneities) and certain critical phenomenon in the NDE of fracture toughness [43,44], this author extended Eshelby's method of equivalent inclusion [119] to the case of time-harmonic case

and presented some preliminary calculations. For the scattering of an ellipsoidal inhomogeneity, details are presented in [120,121]. Equivalence conditions between the inhomogeneity and the inclusion problems are derived and an approximation solution is sought by expanding the eigenstrains in polynomial form. The method is of interest in that the continuity conditions at the matrix-inhomogeneity interface are automatically satisfied. It may have great potential in certain applications if suitable scheme for asymptotic expansion in the evaluation of integrals can be identified [122,123]. Procedures for the inverse scattering problem, appropriate for the Rayleigh limit, are given in [134,135].

Acoustoelasticity and Finite Elasticity

Acoustoelasticity, or acoustical birefringence, is a new area of experimental mechanics which derives its theoretical foundation from the classical treatment of finite deformation in an elastic solid [124,125]. Hughes and Kelly [126] derived expressions for the velocities of elastic waves in stressed solids using Murnaghan's theory and third order terms in the energy. For an isotropic material they showed three higher order constants l , m , and n are needed, in addition to the Lamé constants, to describe the material behavior. When the velocities are measured for given simple state of stress, the Murnaghan's constants l , m and n can be obtained. Results for polystyrene, iron, and pyrex glass were given in [126]. Those for rail steel were reported in [127].

Several acoustic techniques have been developed for the non-destructive evaluation of the stress within an elastic solid where the changes in the propagation speeds of various waves due to changes in stress are measured [64,65,128,129]. In [129], theoretical and acoustical-experimental

results of stress contour plots of the first stress invariant for 6061-T6 aluminum panels, with central hole and with double edge notches, are presented and good agreements are found. The applicability of acoustoelasticity for residual stress determination was studied in [130] for specimens under uniaxial tension loading completely unloaded after yielding occurred. The acoustic response of the material was shown to be somewhat changed due to the plastic flow. It appears that a unifying theory of acousto-effects including elastic-plastic deformations should be developed. Such theory as acousto-elastoplasticity does not seem to be available in current literature.

Considering a homogeneous medium under finite strain in a state of initial stress, Biot [131] discussed the influence of initial stress on elastic waves where he developed the dependence of elastic waves on the initial stress and the density. He also touched upon the question of internal instability related to elastic symmetry. In a recent study Ericksen [132] reviews special topics in elastostatics toward an interpretation from experiment to mechanistic theory of constitutive relations and vice versa. Symmetry-induced instabilities are also discussed. Very interestingly, he pointed out in a subsequent study [133] that the uniqueness condition is violated in some crystals at second order phase transforms involving a change of symmetry and at such some acoustic speeds must vanish but not allowed by the uniqueness condition. If this indicates instability, what consequences develop from this? Does it dictate fracture in materials?

DISCUSSION

The purpose of this review is to bridge the gap among the theoreticians in mechanics and the NDE engineers by focusing on the concepts and available techniques related to the NDT of strength and fracture properties. The questions here are not only the technique in determining these properties but what constitutes the critical condition that is intrinsic to the material and how this is reflected in the measurements of NDT. The mechanics solution therefore must encompass direct scattering, inverse scattering plus a search for the critical condition(s). This task is no doubt most important if NDT measurements are to have a scientific foundation in their interpretation and usage.

Standard techniques and automated electronic system are now available for velocity, attenuation and acoustoelastic measurements. Material properties such as elastic moduli and mass density can be obtained with certain confidence. Strength and fracture properties such as yield stress, fracture toughness, fatigue damage, etc., can be found to correlate with certain ultrasonic (acoustic) factors [40,42,45,46,48,49,50, 55]. The critical conditions contained in these measurements are not understood and therefore cannot be used on production line with confidence.

Methods for solving direct scattering problems, single or multiple scattering, are relatively well developed. For low to medium frequency there are the integral equation method [112-121] and the transition matrix approach, [95-103] and for the high frequency regime there is the geometric diffraction method [76-84]. The ray theory for the study of acoustic emission signals in the time domain has had significant recent advances

[86-89,91-93]. Due to the complexity of the problem and the numerous parameters involved, numerical displays of the computational results are often required.

This may be the very reason why procedures on the inverse problem are available only when the solution can be highly simplified [83,84,93,134,135]. It is widely recognized, however, better fracture mechanics models are needed and should be developed to facilitate automatic electronic inspection of defects in structures. For the detection of sub-surface or embedded cracks, the ray theories have proved to be useful [80,83,84,92,93] and for surface cracks the ray theory [81-83], the acoustoelasticity approach [55] and the dynamic photoelasticity [68] may be fruitful.

Characterization of critical conditions for non-destructive evaluation of strength and fracture properties is an area of vital importance with only limited development so far. The two-grain interaction model proposed in [43,44] is of fundamental interest and yet the required calculation at wavenumber up to $ka \sim 2\pi$ is costly and available only for simple cases, e.g. two spherical holes rather than elastic inclusions. The analysis given in [44] applies to the case where mass-density mismatch is not present such as in the two-phase titanium. The ray theory [91,92], the transition matrix [97-99,102,103], the integral equations method [114-121], and the finite element method [136,137] can be used. Efficient algorithm in programming is of importance here.

The polycrystalline alloys can certainly be considered as composite materials. It appears that there is some evidence in finite elasticity [132,138] that points to instability condition in crystalline materials.

Fundamental insight on the critical conditions that lead to the limit of material strength and fracture toughness may be gained. On the other hand the knowledge of a single inhomogeneity in a polycrystalline environment [139] may offer an as fruitful attractive approach. Developments along these lines are only budding.

Experimental techniques in acoustic emission, acoustoelasticity and ultrasonics have current capabilities in detecting damage and fracture. These are however knowledges and skills that are not generally available nor generally practiced on production line. A full fledged effort in bringing this about requires the integration of electronics, science and engineering, so that the advanced technology that was developed through last decade may be brought to use. The input of mechanics into the coming age of non-destructive evaluation is necessary and must be given sufficient momentum to overcome the hurdles and difficulties in this complex task.

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16. Abstract The scattering of a single ellipsoidal inhomogeneity is studied via an eigenstrain approach. The displacement field is given in terms of volume integrals that involve eigenstrains that are related to mis-match in mass density and that in elastic moduli. The governing equations for these unknown eigenstrains are derived. Agreement with other approaches for the scattering problem is shown. The formulation is general and both the inhomogeneity and the host medium can be anisotropic. The axisymmetric scattering of an ellipsoidal inhomogeneity in a linear elastic isotropic medium is given as an example. The angular and frequency dependence of the scattered displacement field, the differential and total cross-sections are formally given in series expansions for the case of uniformly distributed eigenstrains.			
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